Correlation functions in the $1+1$ dimensional Sine-Gordon field theory

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Abstract of the thesis:

The construction of a genuine, non-perturbative, rigorously defined and truly interacting quantum field theory is a very hard problem on which only scant progress could have been made. It is important to recall that bringing such a program to an end means to be able to characterise, in an explicit and manageable way, all the correlation functions of the theory. In other words, to be able to provide closed formulae for the model's correlation functions -*viz.* the physically observable quantities- and to be able to extract from these all the physically interesting information. Achieving the full program, even for some specific instance of a non-trivial, *viz.* interacting, quantum field theory, would help to forge a deeper understanding, on rigorous ground, of quantum field theories beyond the constructive renormalisation scheme. In particular, it could shed light on new tools of analysis, alternative to the renormalisation be it the constructive or formal one, allowing one to study such theories.

Following the pioneering work of Karowski, Weisz [3] and Kirillov, Smirnov [4], there emerged an efficient technique, called the bootstrap program, allowing one to compute explicitly the correlation functions in numerous completely integrable massive quantum integrable 1+1 dimensional field theories [5]. More precisely, by relying on the factorisability of the multipatricle scattering into two-particle processes which is present in such models, the bootstrap program allows one to provide explicit, multiple integral representations for the matrix elements of local operators $\mathcal{O}(0,0)$ taken between the asymptotic states of the theory

$$ {}_{\mathrm{out}}\langle (\beta_1,\mu_1),\dots,(\beta_n,\mu_n)\,|\,\mathcal{O}(0,0)\,|\,(\alpha_1,\nu_1),\dots,(\alpha_m,\nu_m)\,\rangle_{\mathrm{in}} \; . $$

Here, β_k, resp. α_k, label the rapidities of the asymptotic particles and μ_k, respectively ν_k, the associated internal degrees of freedom. By using the completeness of the asymptotic states one may then compute any multi-point function by means of the form factor expansion. For instance, within the method, a vacuum-to-vacuum two-point function is expressed as

$$ \langle \mathcal{O}^{\dagger}(x,t)\mathcal{O}(0,0)\rangle \;=\; \sum_{n\geq 0}\frac{1}{n!}\int_{\mathbb{R}^n}\mathrm{d}^n\beta \sum_{\mu_1,\dots,\mu_n}\Big|\langle 0\,|\,\mathcal{O}(0,0)\,|\,(\beta_1,\mu_1),\dots,(\beta_n,\mu_n)\,\rangle_{\mathrm{in}}\Big|^2\cdot\prod_{a=1}^{n}\Big\{\mathrm{e}^{\mathrm{i}xp_{\mu_k}(\beta_k)-\mathrm{i}t\varepsilon_{\mu_k}(\beta_k)}\Big\} \; . $$

Above, $p_{\mu_k}(\beta)$ and $\varepsilon_{\mu_k}(\beta)$ are the momentum and energy of the asymptotic particles. While constituting a certain form of an answer, the results provided by the bootstrap program cannot be considered as the end of the story. Indeed, to fully solve the theory, one should develop techniques so as to characterised the above two-point functions explicitly. In this respect, there are several open challenging questions. First of all, the well-definiteness of such representation, and more importantly the proof of the locality of the so-constructed operators, relies on the assumption of convergence of such series [5]. Second, on physical grounds, one is mostly interested in the long-time and/or large-distance behaviour of such operators. It is unclear how to access to such regimes directly from the above expansion.

The aim of the thesis would be to develop techniques allowing one to settle the above questions in the case of the simplest yet non-trivial massive integrable quantum field theory, the so-called 1+1 dimensional Sine-Gordon model [1, 2, 5]. Namely,

i) to prove the convergence of the form factor expansion in this model. Since the form factors $\langle 0 \,|\, \mathcal{O}(0,0) \,|\, (\beta_1, \mu_1), \ldots, (\beta_n, \mu_n) \,\rangle$ admit n-fold multiple integral representations having some structural similarities with the random matrix models issued integrals, this part will demand to extend and generalise various technique appearing in the random matrix theory and allowing one to estimate the large n behaviour of various classes of n-dimensional integrals.

ii) to develop techniques allowing one to extract the sort-distance asymptotic behaviour of two-point functions in the model which is the regime of greatest interest to the physics of the model from the point of view of the universal structure of the model.

It is expect that, on top of the physical spin-offs, the advance on problems $i) - ii)$ will also make important contribution to the modern theory of special function, especially relatively to building new classes of functions lying above the Painlevé transcendents.

The main technical tools that will be handled during the PhD thesis will belong to the field of complex analysis, special functions and probability theory. Also, the research carried out would demand a great deal of calculations.

References

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